

11-7 Multiplying, Dividing, and Simplifying Radicals

Objective: To simplify products and quotients of radicals.

Vocabulary

Rationalize the denominator The process of eliminating a radical from the denominator of a fraction. Remember that $(\sqrt{a})^2 = a$.

Simplest form of a square-root radical

When all of the following are true:

1. No integral radicand has a perfect-square factor other than 1.

2. No fractions are under a radical sign.

3. No radicals are in a denominator.

Simplest form

$$2\sqrt{5}$$

$$\frac{\sqrt{3}}{3}$$

$$\frac{5\sqrt{2}}{2}$$

Not in simplest form.

$$\sqrt{20}$$

$$\sqrt{\frac{1}{3}}$$

$$\frac{5}{\sqrt{2}}$$

Example 1 Simplify $2\sqrt{3} \cdot 3\sqrt{48}$.

$$\begin{aligned} \text{Solution} \quad 2\sqrt{3} \cdot 3\sqrt{48} &= (2 \cdot 3)(\sqrt{3} \cdot \sqrt{48}) \\ &= 6\sqrt{144} \\ &= 6 \cdot 12 \\ &= 72 \end{aligned}$$

Simplify.

1. $6\sqrt{2} \cdot 3\sqrt{2}$

2. $3\sqrt{5} \cdot 2\sqrt{5}$

3. $\sqrt{2} \cdot \sqrt{2} \cdot \sqrt{9}$

4. $\sqrt{3} \cdot \sqrt{3} \cdot \sqrt{16}$

5. $2\sqrt{3} \cdot \sqrt{5}$

6. $4\sqrt{2} \cdot \sqrt{3}$

7. $\sqrt{2} \cdot \sqrt{32}$

8. $\sqrt{3} \cdot \sqrt{27}$

9. $\sqrt{11} \cdot \sqrt{99}$

10. $\sqrt{8} \cdot \sqrt{18}$

11. $4\sqrt{108}$

12. $7\sqrt{80}$

Example 2 Simplify $\sqrt{\frac{7}{6}} \cdot \sqrt{\frac{54}{28}}$.

$$\text{Solution} \quad \sqrt{\frac{7}{6}} \cdot \sqrt{\frac{54}{28}} = \sqrt{\frac{7}{6} \cdot \frac{54}{28}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$$

Simplify.

13. $\sqrt{\frac{7}{10}} \cdot \sqrt{\frac{10}{7}}$

14. $\sqrt{\frac{5}{3}} \cdot \sqrt{\frac{3}{20}}$

15. $\sqrt{\frac{24}{11}} \cdot \sqrt{\frac{33}{2}}$

16. $\sqrt{\frac{7}{5}} \cdot \sqrt{\frac{5}{28}}$

17. $\sqrt{\frac{3}{8}} \cdot \sqrt{\frac{8}{27}}$

18. $\sqrt{\frac{3}{5}} \cdot \sqrt{\frac{125}{3}}$

19. $\sqrt{\frac{7}{3}} \cdot \sqrt{\frac{3}{112}}$

20. $\sqrt{\frac{2}{5}} \cdot \sqrt{\frac{10}{8}}$

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Example 3 Simplify: a. $\frac{2}{\sqrt{3}}$ b. $\sqrt{\frac{5}{8}}$ c. $\frac{5\sqrt{2}}{\sqrt{12}}$ d. $\sqrt{4\frac{4}{5}} \cdot \sqrt{3\frac{1}{3}}$

Solution

a. $\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{(\sqrt{3})^2} = \frac{2\sqrt{3}}{3}$

b. $\sqrt{\frac{5}{8}} = \frac{\sqrt{5}}{\sqrt{8}} = \frac{\sqrt{5}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{5 \cdot 2}}{2(\sqrt{2})^2} = \frac{\sqrt{10}}{4}$

c. $\frac{5\sqrt{2}}{\sqrt{12}} = \frac{5\sqrt{2}}{\sqrt{2^2 \cdot 3}} = \frac{5\sqrt{2}}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{6}}{2(\sqrt{3})^2} = \frac{5\sqrt{6}}{6}$

d. $\sqrt{4\frac{4}{5}} \cdot \sqrt{3\frac{1}{3}} = \sqrt{\frac{24}{5}} \cdot \sqrt{\frac{10}{3}} = \sqrt{\frac{24}{5} \cdot \frac{10}{3}} = \sqrt{16} = 4$

Simplify.

21. $\frac{3}{\sqrt{5}}$

22. $\frac{4}{\sqrt{6}}$

23. $\sqrt{\frac{1}{6}}$

24. $\sqrt{\frac{3}{8}}$

25. $\frac{6\sqrt{5}}{\sqrt{80}}$

26. $\frac{2\sqrt{3}}{\sqrt{48}}$

27. $\sqrt{3\frac{3}{4}} \cdot \sqrt{2\frac{2}{3}}$

28. $\sqrt{1\frac{1}{6}} \cdot \sqrt{4\frac{2}{3}}$

Example 4 Simplify $\sqrt{3}(\sqrt{3} - 4)$.

Solution

$$\begin{aligned}\sqrt{3}(\sqrt{3} - 4) &= \sqrt{3} \cdot \sqrt{3} - \sqrt{3} \cdot 4 \\ &= 3 - 4\sqrt{3}\end{aligned}$$

Simplify.

29. $\sqrt{2}(\sqrt{2} - 1)$

30. $\sqrt{6}(5 - \sqrt{6})$

31. $2\sqrt{3}(\sqrt{27} - \sqrt{3})$

32. $3\sqrt{5}(2\sqrt{5} - \sqrt{125})$

Mixed Review Exercises

Solve.

1. $x^2 = 121$

2. $2x^2 - 128 = 0$

3. $16x^2 - 1 = 24$

4. $\frac{1}{c} + \frac{1}{3} = \frac{1}{2}$

5. $\frac{3}{5} = \frac{15}{y}$

6. $\frac{6b - 1}{3b - 1} = \frac{5}{2}$

Simplify.

7. $15x + 2(3x - 5) + 2$

8. $10a + 6 - (6a - 12)$

9. $3(2a - 5) - 4(a - 3)$

10. $(-4cd^2)(-3c^2d)$

11. $-3m + 2 + 9m - 5$

12. $x(x - 1) + (x - 3)(2x - 1)$